



Chapter -4 motion in a plane

Scalar Quantities:-

The quantities which have magnitude only but no direction. For example : mass, length, time, speed , temperature etc.

Vector Quantities :-

The quantities which have magnitude as well as direction and obeys vector laws of addition, multiplication etc. For examples : Displacement, velocity, acceleration, force , momentum etc.

Addition of Vectors :-

- (i) Only vectors of same nature can be added.
- (ii) The addition of two vector A and B is resultant R

$$\vec{R} = \vec{A} + \vec{B}$$

$$\text{And } R = (A^2 + B^2 + 2AB \cos\theta)^{1/2}$$

$$\text{And } \tan \beta = B \sin\theta / (A + B \cos\theta) ,$$

Where θ is the angle between vector A and vector B, And β is the angle which R makes with the direction of A.

- (iii) Vector addition is commutative $A + B = B + A$
- (iv) Vector addition is associative, $A + (B + C) = (A + B) + C$
- (v) R is maximum if $\theta = 0$ and minimum if $\theta = 180 + 0$.

Subtraction of two vectors :-

- (i) Only vector of same nature can be subtracted.
- (ii) Subtraction of B from A = vector addition of A and (-B),



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$$\text{Where } R = [A^2 + B^2 + 2AB \cos(180 - \Theta)]^{1/2}$$

and

$$\tan \beta = B \sin(180 - \Theta) / [A + B \cos(180 - \Theta)]$$

Where Θ is the angle between A and B and β is the angle which R makes with the direction of A.

(iii) Vector addition is commutative $A + B \neq B + A$

(iv) Vector addition is associative, $A + (B + C) \neq (A + B) + C$

Rectangular components of a vector in a plane :-

If A makes an angle Θ with x-axis and A_x and B_y be the rectangular components of A along X-axis and Y- axis respectively.

Then $A = A_x + B_y = y$

Here $A_x = A \cos \Theta$ and $B_y = A \sin \Theta$

Dot product or scalar product :-

The dot product of two vectors A and B, represented by $\cdot$ is a scalar, which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them. If Θ is the smaller angle between A and B, then $A \cdot B = AB \cos \Theta$

$$\vec{A} \cdot \vec{B} = AB \cos \Theta$$

$$(i) \quad \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$(ii) \quad \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$(iii) \quad \text{If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\text{and } \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\text{Then } \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Cross or Vector product :-



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The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector, which is equal to the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and the sine of the smaller angle between them. If θ is the smaller angle between $\vec{A}$ and $\vec{B}$, then $|\vec{A} \times \vec{B}| = AB \sin \theta$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

$$(i) \quad \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$(ii) \quad \begin{aligned} \hat{i} \times \hat{j} &= \hat{k} & \hat{j} \times \hat{k} &= \hat{i} & \hat{k} \times \hat{i} &= \hat{j} \\ \hat{j} \times \hat{i} &= -\hat{k} & \hat{k} \times \hat{j} &= -\hat{i} & \hat{i} \times \hat{k} &= -\hat{j} \end{aligned}$$

$$(iii) \quad \text{If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad \text{and} \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = (A_x B_z - A_z B_x) \hat{i} + (A_z B_y - A_y B_z) \hat{j} + (A_y B_x - A_x B_y) \hat{k}$$

Projectile motion :-

Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

CIRCULAR MOTION

Circular Motion –

When a body moves such that it always remains at a fixed distance from a fixed point then its motion is said to be circular motion. The fixed distance is called the radius of the circular path and the fixed point is called the center of the circular path.

Uniform circular motion:-

When an object follows a circular path at constant speed. The motion of the object is called uniform circular motion.

The magnitude of its acceleration is $a_c = v^2/R$.

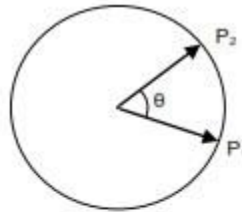
The direction of a_c is always towards the centre of the circle.

Angular Displacement –

Angle swept by the radius vector of a particle moving on a circular path is known as angular displacement of the particle. Example :- angular displacement of the particle from P_1 to P_2 is θ .



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Relation Between Angular Displacement and Linear Displacement –

Since, $\text{Angle} = \frac{\text{arc}}{\text{radius}}$

Angular Displacement = $\frac{\text{arc } P_1P_2}{\text{radius}}$

$$\theta = \frac{s}{r}$$

Angular speed :-

The angular speed is the rate of change of angular distance.

It is related velocity v by $V = \omega R$. The acceleration is $a_c = \omega^2 R$.

If T is the time period of revolution of the object in circular motion and ν is the frequency then we have $\omega = 2\pi\nu$ and acceleration $a = 4\pi^2 \nu^2 R$

$$\omega_{\text{avg}} = \frac{\text{Total Angular Displacement}}{\text{Total Time Taken}}$$

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Velocity –

Angular velocity of a body at some particular instant of time is known as instantaneous angular velocity.

Or

Average angular velocity evaluated for very short duration of time is known as instantaneous angular velocity.



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$$\omega = \lim_{\Delta t \rightarrow 0} \omega_{\text{avg}} = \frac{\Delta \theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

Relation Between Angular Velocity and Linear Velocity

We know that angular velocity

$$\omega = \frac{d\theta}{dt}$$

Putting, $\theta = s/r$

$$\omega = \frac{d(s/r)}{dt}$$

or,

$$\omega = \frac{1}{r} \frac{ds}{dt}$$

or,

$$\omega = \frac{v}{r}$$

or,

$$v = r\omega$$

Time Period of Uniform Circular Motion –

Total time taken by the particle performing uniform circular motion to complete one full circular path is known as time period. In one time period total angle rotated by the particle is 2π and time period is T . Hence angular velocity

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

Frequency -

Number of revolutions made by the particle moving on circular path in one second is known as frequency.

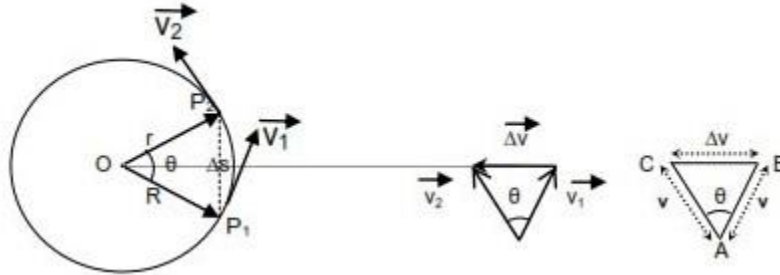
$$f = 1/T = \omega/2\pi$$



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Centripetal Acceleration –

When a body performs uniform circular motion its speed remains constant but velocity continuously changes due to change of direction. Hence a body is continuously accelerated and the acceleration experienced by the body is known as centripetal acceleration (that is the acceleration directed towards the center).



Consider a particle performing uniform circular motion with speed v . When the particle changes its position from P_1 to P_2 , its velocity changes from v_1 to v_2 due to change of direction. The change in velocity from P_1 to P_2 is Δv which is directed towards the center of the circular path according to triangle law of subtraction of vectors.

From figure $\triangle OP_1P_2$ and $\triangle ABC$ are similar,
hence applying the condition of similarity

$$\frac{BC}{AB} = \frac{P_1P_2}{OP_1}$$

or,

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

or,

$$\Delta v = \frac{v \Delta s}{r}$$

Dividing both sides by Δt ,

$$\frac{\Delta v}{\Delta t} = \frac{v \Delta s}{r \Delta t}$$

Taking limit $\Delta t \rightarrow 0$ both sides,

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$$

or,

$$\frac{dv}{dt} = \frac{v ds}{dt}$$

or,

$$a = \frac{v^2}{r}$$

Putting $v = r\omega$,

$$a = r\omega^2$$



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Since the change of velocity is directed towards the center of the circular path, the acceleration responsible for the change in velocity is also directed towards center of circular path and hence it is known as centripetal acceleration.

Centripetal force :-

Centripetal force is the name given to the force that provides inward radial acceleration to a body in circular motion. We should always look for some material force like tension, gravitational force, electrical force, friction etc. as the centripetal force.

Physical Application of Centripetal Force

- i) Case - 1 Circular motion of a stone tied to a string. Centripetal force is provided by the tension of the string
- ii) Case - 2 Circular motion of electron around the nucleus. Centripetal force is provided by the electrostatic force of attraction between the positively charged nucleus and negatively charged electron
- iii) Case - 3 Circular motion of planets around sun or satellites around planet. Centripetal force is provided by the gravitational force of attraction between the planet and sun
- iv) Case - 4 Circular motion of vehicles on a horizontal road. Centripetal force is provided by the static frictional force between the road and the tyre of the vehicle.
- vi) Case - 6 Circular motion of mud particles sticking to the wheels of the vehicle. Centripetal force is provided by the adhesive force of attraction between the mud particles and the tyres of the vehicle. At very high speed when adhesive force is unable to provide necessary centripetal force, the mud particles fly off tangentially. In order to prevent the particles from staining our clothes, mud-guards are provided over the wheels of vehicle.
- vii) Case - 7 Circular motion of a train on a horizontal track. Centripetal force is provided by the horizontal component of the reaction force applied by the outer track on the inner projection of the outer wheels