

Exercise 15

15. Quadrilaterals

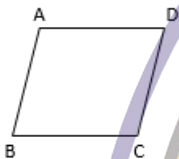
1. Question

Fill in the blanks:

- (i) A quadrilateral hassides.
- (ii) A quadrilateral has..... angles.
- (iii) A quadrilateral has..... vertices, no three of which are.....
- (iv) A quadrilateral hasdiagonals.
- (v) A diagonal of a quadrilateral is a line segment that joins two..... vertices of the quadrilateral.
- (vi) The sum of the angles of a quadrilateral is

Answer

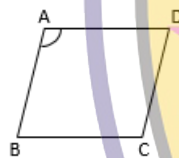
- (i) Four



AB, BC, CD and DA are four sides of this quadrilateral

A quadrilateral is polygon having four sides and four corners.

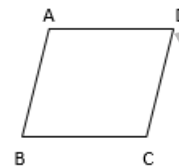
- (ii) four



$\angle A$, $\angle B$, $\angle C$ and $\angle D$ are four angles of this quadrilateral

A quadrilateral is polygon having four sides and four corners.

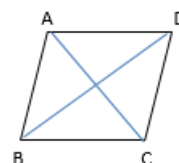
- (iii) Four, collinear



A, B, C and D are the four vertices of this quadrilateral.

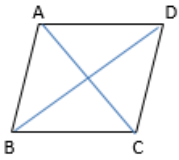
In quadrilateral, no three out of four vertices are collinear. If all the vertices are collinear then we will get a line segment and if three out of four vertices is collinear, we will get a triangle.

- (iv) two



A diagonal is a line segment that joins two opposite vertices of the quadrilateral.

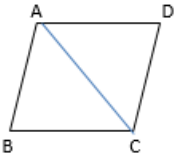
(v) opposite



A diagonal is a line segment that joins two opposite vertices of the quadrilateral.

AC and BD are the two diagonals of the quadrilateral ABCD.

(vi) 360°



ABCD is a quadrilateral and AC is a diagonal. Now, we get two triangles

$\triangle ABC$ and $\triangle ACD$.

As we know that sum of angles of triangle is 180°

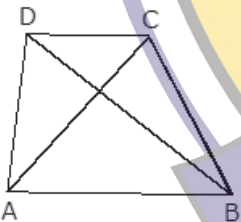
So, sum of two triangles will be $180^\circ \times 2 = 360^\circ$

i.e., the sum of the angles of a quadrilateral is 360°

2. Question

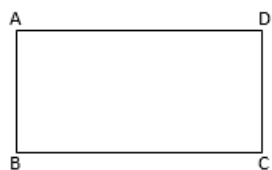
In the adjoining figure, ABCD is a quadrilateral.

- (i) How many pairs of adjacent sides are there? Name them.
- (ii) How many pairs of opposite sides are there? Name them.
- (iii) How many pairs of adjacent angles are there? Name them.
- (iv) How many pairs of opposite angles are there? Name them.
- (v) How many diagonals are there? Name them.



Answer

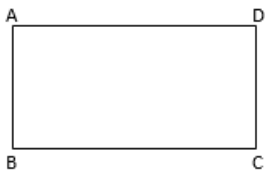
- (i) four; (AB, BC), (BC, CD), (CD, DA), (DA, AB)



When two sides of quadrilateral have same end point, they are called as Adjacent Sides.

(AB, BC), (BC, CD), (CD, DA), (DA, AB) are the adjacent sides of this quadrilateral.

- (ii) two; (AB, DC), (AD, BC)



Two sides of quadrilateral who do have same end point are called as Opposite Sides.

(AB, DC) , (AD, BC) are the opposite sides of this quadrilateral.

(iii)

four; $(\angle A, \angle B)$, $(\angle B, \angle C)$,

$(\angle C, \angle D)$, $(\angle D, \angle A)$



When two angles of quadrilateral share the common arm it is called as Adjacent angles of the quadrilateral.

$(\angle A, \angle B)$, $(\angle B, \angle C)$, $(\angle C, \angle D)$ and $(\angle D, \angle A)$ are adjacent angles of this quadrilateral.

(iv)

two; $(\angle A, \angle C)$, $(\angle B, \angle D)$



When two angles of quadrilateral are not adjacent angles then it is called as opposite angles of the quadrilateral.

$(\angle A, \angle C)$ and $(\angle B, \angle D)$ are opposite angles of this quadrilateral.

(v) two; (AC, BD)

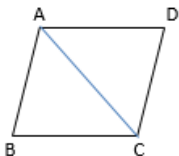
A diagonal is a line segment that joins two opposite vertices of the quadrilateral.

AC and BD are the two diagonals of the quadrilateral $ABCD$.

3. Question

Prove that the sum of the angles of a quadrilateral is 360° .

Answer



$ABCD$ is a quadrilateral and AC is a diagonal. Now, we get two triangles

$\triangle ABC$ and $\triangle ACD$.

As we know that sum of angles of triangle is 180°

So, sum of two triangles will be $180^\circ \times 2 = 360^\circ$

i.e. the sum of the angles of a quadrilateral is 360°

4. Question

The three angles of a quadrilateral are 76° , 54° and 108° . Find the measure of the fourth angle.

Answer

Let $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are the four angles of quadrilateral.

As we know that, Sum of all four angles of quadrilateral is 360° .

$$\angle A = 76^\circ$$

$$\angle B = 54^\circ$$

$$\angle C = 108^\circ$$

$$\text{So, } \angle D = 360^\circ - (\angle A + \angle B + \angle C)$$

$$= 360^\circ - (76^\circ + 54^\circ + 108^\circ)$$

$$= 122^\circ$$

So, fourth angle of quadrilateral will be 122° .

5. Question

The angles of a quadrilateral are in the ratio 3 : 5 : 7 : 9. Find the measure of each of these angles.

Answer

Let x be the common multiple.

As per question,

$$\angle A = 3x$$

$$\angle B = 5x$$

$$\angle C = 7x$$

$$\angle D = 9x$$

As we know that, Sum of all four angles of quadrilateral is 360° .

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$3x + 5x + 7x + 9x = 360^\circ$$

$$24x = 360^\circ$$

$$x = 360/24$$

$$= 15^\circ$$

$$\angle A = 3 \times 15^\circ = 45^\circ$$

$$\angle B = 5 \times 15^\circ = 75^\circ$$

$$\angle C = 7 \times 15^\circ = 105^\circ$$

$$\angle D = 9 \times 15^\circ = 135^\circ$$

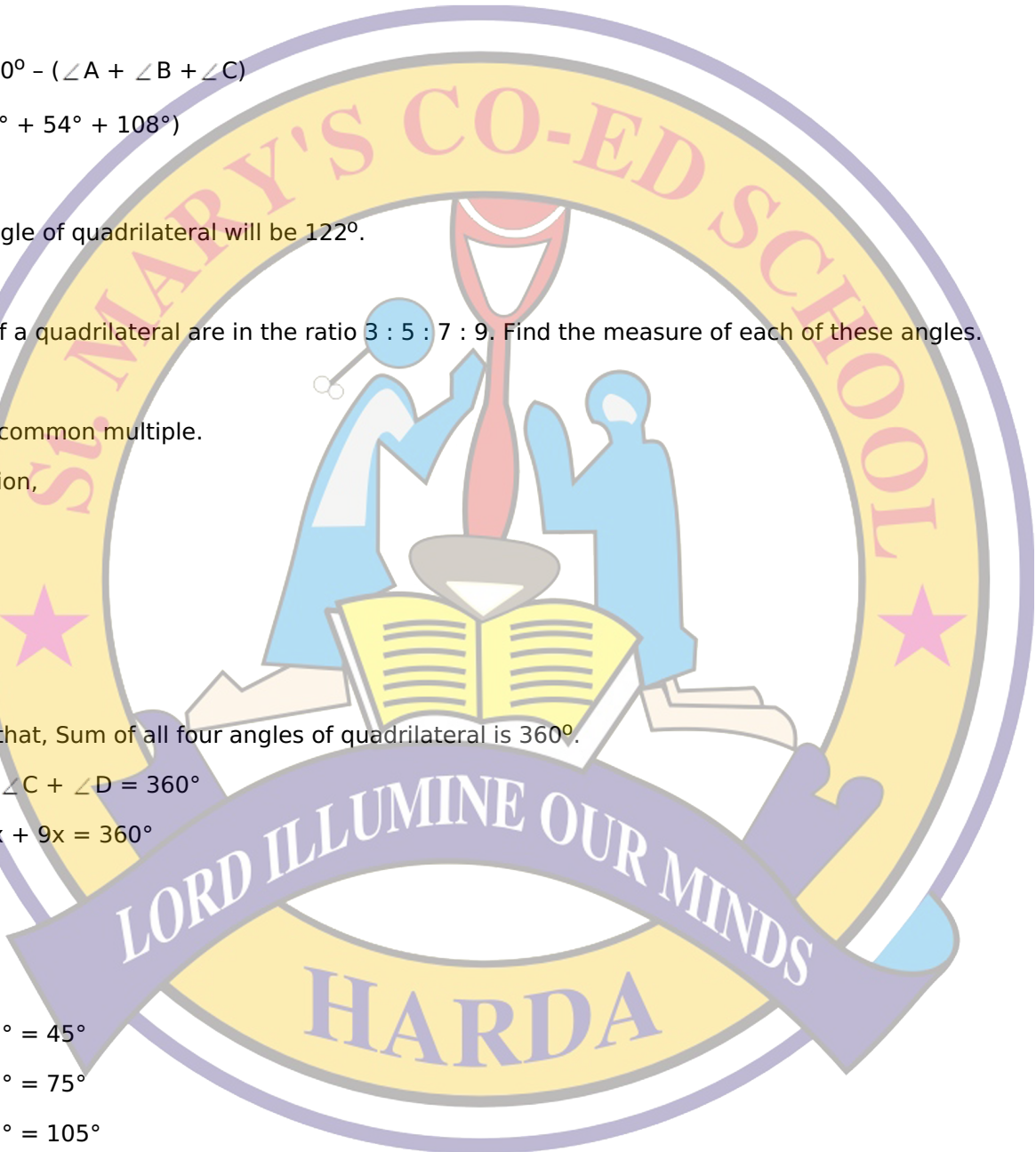
So, Angles of quadrilateral are 45° , 75° , 105° and 135° .

6. Question

A quadrilateral has three acute angles, each measuring 75° . Find the measure of the fourth angle.

Answer

Three angles are acute angle and each measuring is 75° means



(Acute angle is angle whose measuring is greater than 0 and less than 90.)

As we know that, Sum of all four angles of quadrilateral is 360° .

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$75^\circ + 75^\circ + 75^\circ + \angle D = 360^\circ$$

$$\angle D = 360^\circ - (75^\circ + 75^\circ + 75^\circ)$$

$$= 360^\circ - 225^\circ$$

$$= 135^\circ$$

So, fourth angle of quadrilateral is 135° .

7. Question

Three angles of a quadrilateral are equal and the measure of the fourth angle is 120° . Find the measure of each of the equal angles.

Answer

Let x be the common angle of quadrilateral.

As per question,

$$\angle A = \angle B = \angle C = x$$

$$\angle D = 120^\circ$$

As we know that, Sum of all four angles of quadrilateral is 360° .

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$x + x + x + 120^\circ = 360^\circ$$

$$3x = 360^\circ - 120^\circ$$

$$3x = 240^\circ$$

$$x = 240 / 3$$

$$= 80^\circ$$

$$\angle A = \angle B = \angle C = 80^\circ$$

So, Three Angles of quadrilateral whose measuring's are equal is 80° .

8. Question

Two angles of a quadrilateral measure 85° and 75° respectively. The other two angles are equal. Find the measure of each of these equal angles.

Answer

Let x be the common angle of quadrilateral.

As per question,

$$\angle A = 85^\circ$$

$$\angle B = 75^\circ$$

$$\angle C = \angle D = x$$

As we know that, Sum of all four angles of quadrilateral is 360° .

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$85^\circ + 75^\circ + x + x = 360^\circ$$

$$2x = 200^\circ$$

$$x = 200 / 2$$

$$= 100^\circ$$

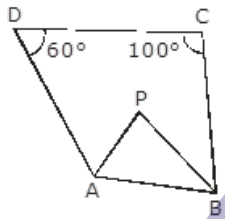
$$\angle C = \angle D = 100^\circ$$

So, Two angles of quadrilateral whose measuring's are equal is 100° .

9. Question

In the adjacent figure, the bisectors of $\angle A$ and $\angle B$ meet in a point P. D

If $\angle C = 100^\circ$ and $\angle D = 60^\circ$, find the measure of $\angle APB$.



Answer

As we know that, Sum of all four angles of quadrilateral is 360° .

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\angle A + \angle B + 100^\circ + 60^\circ = 360^\circ$$

$$\angle A + \angle B = 360^\circ - 160^\circ$$

$$= 200^\circ$$

Now, according to question bisector of $\angle A$ and $\angle B$ meet in a point P and forms the triangle PAB.

So,

$$\frac{1}{2} \angle A + \frac{1}{2} \angle B = 200^\circ / 2$$

$$= 100^\circ$$

As we know that, sum of all angles of triangle is 180° .

$$\angle BAP + \angle ABP + \angle APB = 180^\circ$$

$$\frac{1}{2} \angle A + \frac{1}{2} \angle B + \angle APB = 180^\circ$$

$$100^\circ + \angle APB = 180^\circ$$

$$\angle APB = 180^\circ - 100^\circ$$

$$= 80^\circ$$

So, $\angle APB = 80^\circ$